$C_n(q,t)$

$$R = \mathbb{C}[x_1, x_2, \dots, x_n, y_1, \dots, y_n] \circlearrowleft S_n$$

$$R \big/ \langle R_+^{S_n} \rangle \qquad \text{diagonal coinvariants}$$

$$V = \{ f \in \perp \mid w.f = \text{sign}(w) f \quad \forall w \in S_n \}$$

$$f = \prod_{i<j} (x_i - x_j)$$

<u>Th'm</u>

$$C_n(q,t) = \sum_{i,j} \dim_{\mathbb{C}} (V_{\substack{\deg x = i \\ \deg y = j}}) \, q^i t^j$$

$$P = \mathbb{C}[x_1, \dots, x_n] \circlearrowleft S_n$$

coinvariant ring $P \big/ \langle P_+^{S_n} \rangle \cong P \big/ \langle e_1(x_1, \dots, x_n), \dots \rangle$

$\dim = n!$

$\dim_q = [n]_q!$

$$\mathrm{Gr}(k,n) = \{ V \subset \mathbb{C}^n \mid \dim V = k \}$$

$$\begin{array}{cc} k & \begin{bmatrix} | & | & & | \\ c_1 & c_2 & \cdots & c_n \\ | & | & & | \end{bmatrix} \\ & n \end{array} \qquad \mathrm{rowspan}(V)$$

$$\mathring{\Pi}(k,n) \subset \mathrm{Gr}(k,n)$$

$$\begin{aligned} &\mathring{\Pi}(k,n) = \{ V \mid \det(c_1 \cdots c_k) \neq 0,\ \det(c_2 \cdots c_{k+1}) \neq 0 \ \ldots \\ &\qquad \det(c_n c_1 \cdots c_{k-1}) \neq 0 \} \end{aligned}$$

$$T \subset PGL(n) \qquad T \circlearrowright \mathrm{Gr}(k,n)$$

$$T \cong (\mathbb{C}^*)^{n-1}$$ scaling columns.

$$X(k,n) := \mathring{\Pi}(k,n) / (\mathbb{C}^*)^{n-1}$$ Catalan variety

Th'm
- $\dim H^*(X(k,n)) = C_{k,n-k}$.
- $C_{k,n-k}(q,1) \doteq$ Poincare poly

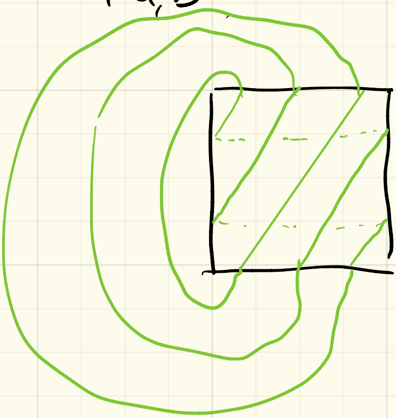
• $\mathcal{X}(k,n)(\mathbb{F}_q) = q^{?}\, C_{k,n-k}\left(q, \frac{1}{q}\right)$

$$\mathcal{X}(k,n) \cong \coprod (\mathbb{C})^{y} \times (\mathbb{C}^{*})^{x}$$

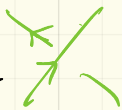
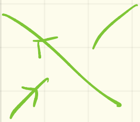
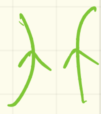
$\#$ maximal Deograms $=$ Euler char $(\mathcal{X}(k,n))$

Knot homology

$T_{a,b}$ torus knot



slope $= \frac{3}{2}$

q  $- q^{-1}$  $= (z - z^{-1})$ 

Th'm $\quad \mathrm{HOMFLY}(T_{a,b}) = \frac{1}{[a+b]_q} \begin{bmatrix} a+b \\ a \end{bmatrix}_q + \cdots$

HOMFLY $\rightsquigarrow$ "KR homology"

Th'm $\quad KR(T_{a,b}) = C_{ab}(q,t) + \cdots$

$$V \subset \mathbb{F}_q^n$$

$$\begin{bmatrix} 1 & * & 0 & * & 0 & * & * \\ & & 1 & * & 0 & * & * \\ & & & & 1 & * & * \end{bmatrix}$$

$$\begin{bmatrix} * & * & * & 0 & * & * & 0 & 1 \\ * & * & * & 0 & * & * & 1 & \\ * & * & * & 1 & & & & \end{bmatrix}$$

$$\begin{array}{ccc} 1 & 0 & 0 \\ & 1 & 0 \\ & & 1 \end{array}$$