1 Logistics

We agreed to use overleaf.com for working on the final report. Do you have an overleaf account set up?

Drawing diagrams in Latex: standard is the `tikz` package

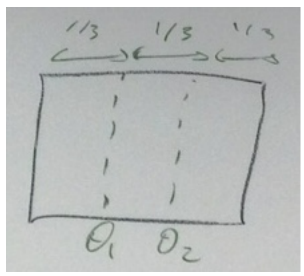
`https://en.wikibooks.org/wiki/LaTeX/PGF/TikZ`

`http://www.texample.net/tikz/examples/ac-drive-voltage/`

Next meeting: Friday February 1, 10 - 11am.

2 Moduli spaces

Problem 1. (a) What is the moduli space of folding configurations for two parallel creases, parallel to opposite sides, which split the sheet into 3 regions of equal area?

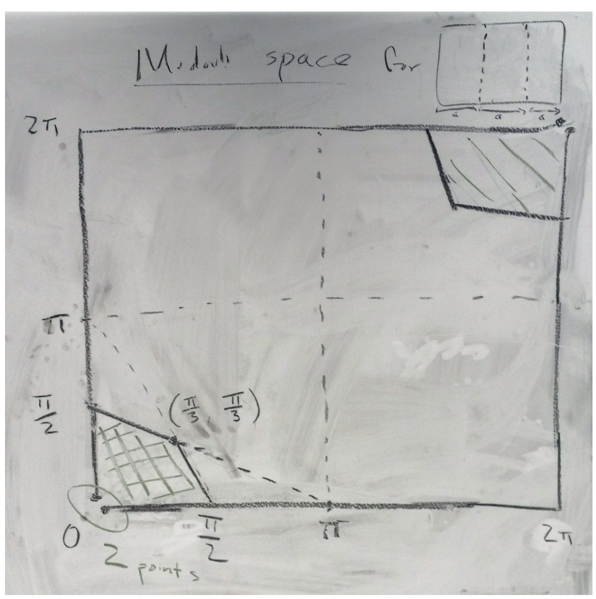


(b) What is the moduli space of folding configurations for two parallel creases, parallel to opposite sides, which are spaced arbitrarily?

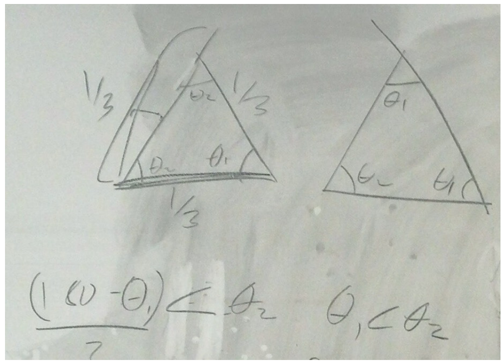
(c) What distance is needed between the creases for this moduli space to look like Example 2 (the "far away" case)? What happens as the distance separating the two creases approaches 0?

Discussion: In the massless case, the space of folding configurations is a torus, for arbitrary placement of the parallel creases.

For Problem 1(a), we described the answer to the massive case explicitly.

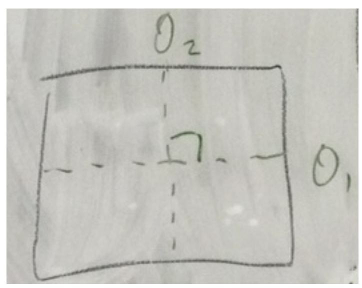


The moduli space is a square with regions removed from two opposite corners. The region is bounded by straight line segments. In addition, two of the extreme corner points become separated into two points each.



For Problem 1(c), if the parallel creases separate the sheet into regions of width a, b, and c, then the "far away" case occurs when $b \geq a + c$.

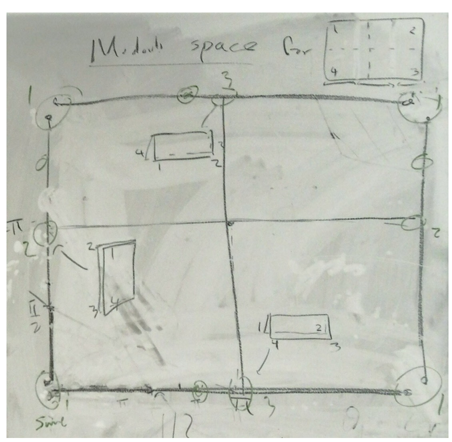
Problem 2. (a) What is the moduli space of folding configurations for two perpendicular creases?



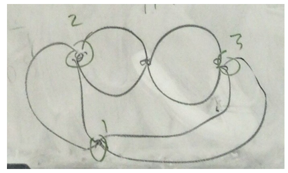
(b) What is the moduli space of folding configurations for two creases which cross in the center at an arbitrary angle? (Assume the creases intersect in the interior of the sheet)?

(c) Does your answer to (a) change if the creases are not aligned with the sides of the sheet? Does your answer to (a) or (b) change if the creases don't intersect at the center?

Discussion: This moduli space has 1 degree of freedom. In the massive case:



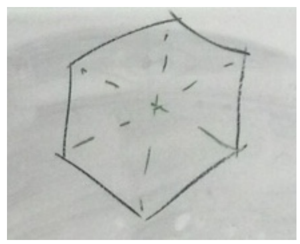
In the massless case:



Problem 3. [non-folding example of a moduli space] What is the moduli space of "space quadrilaterals" with fixed side lengths? E.g. if I cut four lengths of straw (with four different colors) and connect them in a loop with string, what are all possible configurations of this object (up to rotations and translations in 3-space)?

(Thanks to Alex Wright for the straw polygons!)

Problem 4. What is the space of folding configurations of a hexagon with creases along its three axes?



<u>Expectations</u> for next meeting:

- For Problem 3, we agreed that the space quadrilateral has 2 degrees of freedom, i.e. its moduli space is 2-dimensional. What is this space? (What is a "common name" for this shape?)
- Answer Problem 1(b-c) completely, and Problem 2(b-c) completely. On Problem 1, the *law of sines* should be helpful.
- Start thinking about Problem 4, the hexagon. (I think this is more complicated) Figure out at least the number of degrees of freedom of this folding pattern.
- **[Writing]** Write up answers to Problems 1(a) and 2(a) nicely in latex, to use as Examples in your final report.
- **[Visualization]** Can you write code to visualize a space of fold configurations? I.e. for Problem 1 or 2, display the moduli space on one side of the screen and as you move the cursor over this space, display the fold configuration on the other side of the screen.
- In the "Discrete folding" paper, try to understand Figure 9 on p. 10. How are the diagrams in the top row special? Where do the numbers next to each diagram come from? Is the model considered here "massless" or "massive"?